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Quantum Bidirectional Teleportation 2 ↔ 2 or 2 ↔ 3 Qubit Teleportation Protocol Via 6-Qubit Entangled State

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Abstract
In this paper, firstly we propose a new protocol for quantum bidirectional teleportation of two particles in a six-qubit entangled channel, which is based on GHZ measurement and unitary operation. Then, also based on the six-qubit entangled state channel, a protocol for quantum bidirectional teleportation of two-three particles is proposed. The protocol utilizes Toffoli gate to convert three-qubit states into two-qubit states and a single qubit, and the rest of the teleportation is the same as the first protocol. Finally, we introduce an auxiliary bit to restore the state of the sender. Both these proposed protocols are more efficient than others in the same situation, which will be compared at the end of the paper.

Keywords Quantum bidirectional teleportation · Six-qubit entangled channel · GHZ measurement · Toffoli gate

1 Introduction

Quantum teleportation (QT) is a new communication method, and the research on QT is very important to achieve the global quantum communication network. In 1993, Bennett et al. [1] proposed the transmission protocol of QT between Alice and Bob. Over the past years, more and more QT protocols have been proposed. Some of them are unidirectional QT [2–4], and some are quantum bidirectional teleportation (QBT) [5–7]. The number of particles of the channel can be different in two protocols, such as four-qubit channel [8–10]、five-qubit channel [11–13]、six-qubit channel [14–16]、seven-qubit channel [17–19] and so on. In

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order to enhance the security of the teleportation, some protocols add a supervisor Charlie to control the teleportation [20].

There have been many protocols for quantum teleportation since 1993. How to improve the efficiency in same teleportation situation is crucial. In this paper, two protocols for 2 ↔ 2 and 2 ↔ 3 qubit teleportation via six-qubit entangled state are proposed. In the first protocol, Alice and Bob want to send a two-particle entangled state to each other. They do a GHZ measurement with one particle of the channel respectively. Then, Alice and Bob inform each other of her/his measurement result by the classical channel. Finally, they perform unitary transformations to restore the target state. In the second protocol, Alice are two-particle entangled state, Bob are three-particle entangled state. In this protocol, we utilize a Toffoli gate with the first two particle of Bob's state as the control qubits and the third particle as the target bit. Then, Bob's state becomes a two-qubit entangled state and a single qubit. The rest of operations are the same as the first protocol. But in the end, an auxiliary particle is introduced to restore Bob's state by perform Toffoli gate again with two of Alice's particles.

More details about the two protocols will be given in Section 2. In Section 3, the efficiency of two protocols and other protocols will be compared. Finally, Section 4 is a summary of the whole paper.

2 QBT Protocol Via 6-Qubit Entangled State

In this section, we discuss two methods for QBT protocol. Our scheme is described as follows. There are two legitimate users, Alice and Bob. They are both sender and receiver. We suppose Alice wants to send a two-qubit state $|\psi\rangle_A$, and it's also the target state of Bob. Bob wants to send a two-qubit or three-qubit state $|\psi\rangle_B$, and it's also the target state of Alice.

To start with, $|\psi\rangle_A$, $|\psi\rangle_B$ are defined as

$$\begin{aligned} |\psi\rangle_A &= \alpha|00\rangle_{a_1a_2} + \beta|11\rangle_{a_1a_2}, \\ |\psi\rangle_B &= x|00\rangle_{b_1b_2} + y|11\rangle_{b_1b_2}. \end{aligned} \tag{1}$$

Where α, β, x, y are arbitrary coefficients with $|\alpha|^2 + |\beta|^2 = 1$, $|x|^2 + |y|^2 = 1$.

The quantum channel is a six-qubit entangled state shared by Alice and Bob, the qubits

$$|\phi\rangle_{123456} = \frac{1}{2}\Big(|000000\rangle + |001011\rangle + |110100\rangle + |111111\rangle\Big)_{123456} \tag{2}$$

1, 5, 6 belong to Alice, denoted as A_1, A_2, A_3, qubits 3, 2, 4 belong to Bob, denoted as B_1, B_2, B_3. The joint state of the senders and the quantum channel is

$$|T\rangle = |\psi\rangle_A \otimes |\psi\rangle_B \otimes |\phi\rangle_{A_1B_2B_1B_3A_2A_3}. \tag{3}$$

In the two protocols, we do GHZ measurements on three particles, the GHZ states are described as

$$\begin{aligned} |\xi_0^{\pm}\rangle &= \frac{1}{\sqrt{2}}\Big[|000\rangle \pm |111\rangle\Big], \\ |\xi_1^{\pm}\rangle &= \frac{1}{\sqrt{2}}\Big[|100\rangle \pm |011\rangle\Big], \\ |\xi_2^{\pm}\rangle &= \frac{1}{\sqrt{2}}\Big[|010\rangle \pm |101\rangle\Big], \\ |\xi_3^{\pm}\rangle &= \frac{1}{\sqrt{2}}\Big[|110\rangle \pm |001\rangle\Big]. \end{aligned} \tag{4}$$

But during the measurement, we just use $|\xi_0^{\pm}\rangle$ and $|\xi_3^{\pm}\rangle$ states.

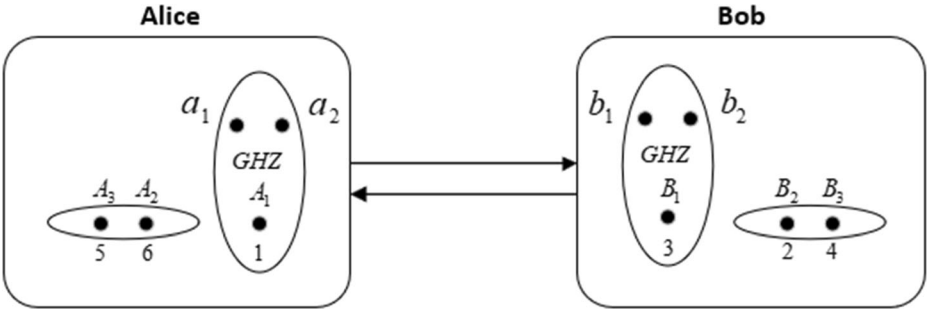

Fig. 1 Schematic of proposed QBT protocol for transmitting the 2 and 2 qubits state between Alice and Bob

2.1 Transmitting the 2 and 2 Qubits State

This scheme is designed to three steps as follows, and the process is illustrated in Fig.1.

Alice and Bob do GHZ measurements on qubit pairs (A, 1) and (B,3), respectively. Then they should tell each other about the measuring result via the classical channel.

In this paper, the state of joint state $|T\rangle$ is

$$\begin{aligned} |T\rangle &= |\psi\rangle_A \otimes |\psi\rangle_B \otimes |\phi\rangle_{A_1B_2B_1B_3A_2A_3} \\ &= \left(\alpha|00\rangle + \beta|11\rangle\right)_{a_1a_2} \otimes \left(x|00\rangle + y|11\rangle\right)_{b_1b_2} \otimes \frac{1}{2}\left(|000000\rangle + |001011\rangle + |110100\rangle + |111111\rangle\right). \end{aligned} \tag{5}$$

After Alice and Bob perform GHZ measurements on three of her/his particles, the rest of the particles A_2,A_3,B_2,B_3 collapse to the states $|\varphi\rangle$. Collapse state $|\varphi\rangle$ corresponding to different measurement results is shown in Table 1, the collapse results have 16 possible states with the same possibility. In Table 1, $|\xi_0^{\pm}\rangle$ and $|\xi_3^{\pm}\rangle$ are GHZ states as mentioned above, and the subscript represents the particles being measured. For ease of expression, we have omitted the multiplier factor 1/4.

Table 1 GHZ measurement results and the corresponding collapsed states of B_2, B_3, A_2, A_3

Alice's results	Bob's results	The collapsed state of qubits B_2, B_3, A_2, A_3
$\|\xi_0^+\rangle_{a_1a_2A_1}$	$\|\xi_0^+\rangle_{b_1b_2B_1}$	$\alpha x \mid 0000\rangle + \beta x \mid 1100\rangle + \alpha y \mid 0011\rangle + \beta y \mid 1111\rangle$
	$\|\xi_0^-\rangle_{b_1b_2B_1}$	$\alpha x \mid 0000\rangle + \beta x \mid 1100\rangle - \alpha y \mid 0011\rangle - \beta y \mid 1111\rangle$
	$\|\xi_3^+\rangle_{b_1b_2B_1}$	$\alpha y \mid 0000\rangle + \beta y \mid 1100\rangle + \alpha x \mid 0011\rangle + \beta x \mid 1111\rangle$
	$\|\xi_3^-\rangle_{b_1b_2B_1}$	$\alpha y \mid 0000\rangle + \beta y \mid 1100\rangle - \alpha x \mid 0011\rangle - \beta x \mid 1111\rangle$
$\|\xi_0^-\rangle_{a_1a_2A_1}$	$\|\xi_0^+\rangle_{b_1b_2B_1}$	$\alpha x \mid 0000\rangle - \beta x \mid 1100\rangle + \alpha y \mid 0011\rangle - \beta y \mid 1111\rangle$
	$\|\xi_0^-\rangle_{b_1b_2B_1}$	$\alpha x \mid 0000\rangle - \beta x \mid 1100\rangle - \alpha y \mid 0011\rangle + \beta y \mid 1111\rangle$
	$\|\xi_3^+\rangle_{b_1b_2B_1}$	$\alpha y \mid 0000\rangle - \beta y \mid 1100\rangle + \alpha x \mid 0011\rangle - \beta x \mid 1111\rangle$
	$\|\xi_3^-\rangle_{b_1b_2B_1}$	$\alpha y \mid 0000\rangle - \beta y \mid 1100\rangle - \alpha x \mid 0011\rangle + \beta x \mid 1111\rangle$
$\|\xi_3^+\rangle_{a_1a_2A_1}$	$\|\xi_0^+\rangle_{b_1b_2B_1}$	$\beta x \mid 0000\rangle + \alpha x \mid 1100\rangle + \beta y \mid 0011\rangle + \alpha y \mid 1111\rangle$
	$\|\xi_0^-\rangle_{b_1b_2B_1}$	$\beta x \mid 0000\rangle + \alpha x \mid 1100\rangle - \beta y \mid 0011\rangle - \alpha y \mid 1111\rangle$
	$\|\xi_3^+\rangle_{b_1b_2B_1}$	$\beta y \mid 0000\rangle + \alpha y \mid 1100\rangle + \beta x \mid 0011\rangle + \alpha x \mid 1111\rangle$
	$\|\xi_3^-\rangle_{b_1b_2B_1}$	$\beta y \mid 0000\rangle + \alpha y \mid 1100\rangle - \beta x \mid 0011\rangle - \alpha x \mid 1111\rangle$
$\|\xi_3^-\rangle_{a_1a_2A_1}$	$\|\xi_0^+\rangle_{b_1b_2B_1}$	$\beta x \mid 0000\rangle - \alpha x \mid 1100\rangle + \beta y \mid 0011\rangle - \alpha y \mid 1111\rangle$
	$\|\xi_0^-\rangle_{b_1b_2B_1}$	$\beta x \mid 0000\rangle - \alpha x \mid 1100\rangle - \beta y \mid 0011\rangle + \alpha y \mid 1111\rangle$
	$\|\xi_3^+\rangle_{b_1b_2B_1}$	$\beta y \mid 0000\rangle - \alpha y \mid 1100\rangle + \beta x \mid 0011\rangle - \alpha x \mid 1111\rangle$
	$\|\xi_3^-\rangle_{b_1b_2B_1}$	$\beta y \mid 0000\rangle - \alpha y \mid 1100\rangle - \beta x \mid 0011\rangle + \alpha x \mid 1111\rangle$

Table 2 Different results and the corresponding unitary operation of Alice and Bob

Alice's results	Bob's results	The collapsed state of qubits B_2, B_3, A_2, A_3	Bob's operations	Alice's operations
$\|\xi_0^+\rangle_{a_1a_2A_1}$	$\|\xi_0^+\rangle_{b_1b_2B_1}$	$(\alpha\|00\rangle+\beta\|11\rangle)(x\|00\rangle+y\|11\rangle)$	$I_{B_2}\otimes I_{B_3}$	$I_{A_2}\otimes I_{A_3}$
	$\|\xi_0^-\rangle_{b_1b_2B_1}$	$(\alpha\|00\rangle+\beta\|11\rangle)(x\|00\rangle-y\|11\rangle)$	$I_{B_2}\otimes I_{B_3}$	$\sigma_{A_2}^Z\otimes I_{A_3}$
	$\|\xi_3^+\rangle_{b_1b_2B_1}$	$(\alpha\|00\rangle+\beta\|11\rangle)(y\|00\rangle+x\|11\rangle)$	$I_{B_2}\otimes I_{B_3}$	$\sigma_{A_2}^X\otimes\sigma_{A_3}^X$
	$\|\xi_3^-\rangle_{b_1b_2B_1}$	$(\alpha\|00\rangle+\beta\|11\rangle)(y\|00\rangle-x\|11\rangle)$	$I_{B_2}\otimes I_{B_3}$	$-i\sigma_{A_2}^Y\otimes I_{A_3}$
$\|\xi_0^-\rangle_{a_1a_2A_1}$	$\|\xi_0^+\rangle_{b_1b_2B_1}$	$(\alpha\|00\rangle-\beta\|11\rangle)(x\|00\rangle+y\|11\rangle)$	$\sigma_{B_2}^Z\otimes I_{B_3}$	$I_{A_2}\otimes I_{A_3}$
	$\|\xi_0^-\rangle_{b_1b_2B_1}$	$(\alpha\|00\rangle-\beta\|11\rangle)(x\|00\rangle-y\|11\rangle)$	$\sigma_{B_2}^Z\otimes I_{B_3}$	$\sigma_{A_2}^Z\otimes I_{A_3}$
	$\|\xi_3^+\rangle_{b_1b_2B_1}$	$(\alpha\|00\rangle-\beta\|11\rangle)(y\|00\rangle+x\|11\rangle)$	$\sigma_{B_2}^Z\otimes I_{B_3}$	$\sigma_{A_2}^X\otimes\sigma_{A_3}^X$
	$\|\xi_3^-\rangle_{b_1b_2B_1}$	$(\alpha\|00\rangle-\beta\|11\rangle)(y\|00\rangle-x\|11\rangle)$	$\sigma_{B_2}^Z\otimes I_{B_3}$	$-i\sigma_{A_2}^Y\otimes I_{A_3}$
$\|\xi_3^+\rangle_{a_1a_2A_1}$	$\|\xi_0^+\rangle_{b_1b_2B_1}$	$(\beta\|00\rangle+\alpha\|11\rangle)(x\|00\rangle+y\|11\rangle)$	$\sigma_{B_2}^X\otimes\sigma_{B_3}^X$	$I_{A_2}\otimes I_{A_3}$
	$\|\xi_0^-\rangle_{b_1b_2B_1}$	$(\beta\|00\rangle+\alpha\|11\rangle)(x\|00\rangle-y\|11\rangle)$	$\sigma_{B_2}^X\otimes\sigma_{B_3}^X$	$\sigma_{A_2}^Z\otimes I_{A_3}$
	$\|\xi_3^+\rangle_{b_1b_2B_1}$	$(\beta\|00\rangle+\alpha\|11\rangle)(y\|00\rangle+x\|11\rangle)$	$\sigma_{B_2}^X\otimes\sigma_{B_3}^X$	$\sigma_{A_2}^X\otimes\sigma_{A_3}^X$
	$\|\xi_3^-\rangle_{b_1b_2B_1}$	$(\beta\|00\rangle+\alpha\|11\rangle)(y\|00\rangle-x\|11\rangle)$	$\sigma_{B_2}^X\otimes\sigma_{B_3}^X$	$-i\sigma_{A_2}^Y\otimes I_{A_3}$
$\|\xi_3^-\rangle_{a_1a_2A_1}$	$\|\xi_0^+\rangle_{b_1b_2B_1}$	$(\beta\|00\rangle-\alpha\|11\rangle)(x\|00\rangle+y\|11\rangle)$	$-i\sigma_{B_2}^Y\otimes I_{B_3}$	$I_{A_2}\otimes I_{A_3}$
	$\|\xi_0^-\rangle_{b_1b_2B_1}$	$(\beta\|00\rangle-\alpha\|11\rangle)(x\|00\rangle-y\|11\rangle)$	$-i\sigma_{B_2}^Y\otimes I_{B_3}$	$\sigma_{A_2}^Z\otimes I_{A_3}$
	$\|\xi_3^+\rangle_{b_1b_2B_1}$	$(\beta\|00\rangle-\alpha\|11\rangle)(y\|00\rangle+x\|11\rangle)$	$-i\sigma_{B_2}^Y\otimes I_{B_3}$	$\sigma_{A_2}^X\otimes\sigma_{A_3}^X$
	$\|\xi_3^-\rangle_{b_1b_2B_1}$	$(\beta\|00\rangle-\alpha\|11\rangle)(y\|00\rangle-x\|11\rangle)$	$-i\sigma_{B_2}^Y\otimes I_{B_3}$	$-i\sigma_{A_2}^Y\otimes I_{A_3}$

To transform the result via classical channel, we encode the 16 possible results in four classical bits. Then Alice and Bob perform unitary operations on all consequences to restore the target state. Their operations are shown in Table 2. In Table 2, σ_m^n is an operation mark, m represents the action object and n represents the operation being performed. I_k represents unit gate transformation, and k represents the action object.

2.2 Transmitting the 2 and 3 Qubits State

Given the teleportation of two and three qubits state, we introduce the Toffoli gate. The Toffoli gate has 3 input qubits and 3 output qubits. As is shown in Fig.2, the first two control qubits are not affected by the Toffoli gate, and the third qubit won't change unless the two control qubits is $|1\rangle$ at the same time.

Step 1. In this protocol, Bob has a three-qubit entangled state as

$$|\psi\rangle_B = x|000\rangle_{b_1b_2b_3} + y|111\rangle_{b_1b_2b_3}, \tag{6}$$

we perform Toffoli gate with b_1, b_2 as the control qubits and b_3 as the target bit. The diagram is shown in Fig.3.

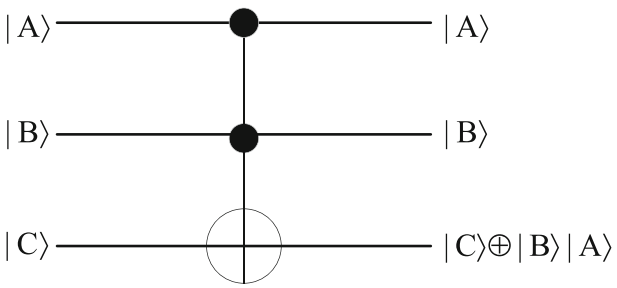

Fig. 2 The Toffoli gate's Circuit representation

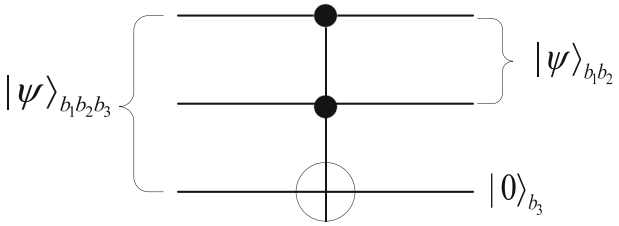

Fig. 3 The diagram of transform Bob's state into a two-qubit state and a single qubit

Then Bob's state $|\psi\rangle_{b_1b_2b_3}$ becomes two-qubit entangled state $|\psi\rangle_{b_1b_2}$ and a single qubit $|0\rangle_{b_3}$.

Step 2. The next step is the transmission of $|\psi\rangle_{a_1a_2}$ and $|\psi\rangle_{b_1b_2}$, it's the same process as the first protocol. Firstly, Alice and Bob do GHZ measurements on qubit pairs (a_1,a_2, 1) and (b_1,b_2,3), respectively. Then, Alice and Bob perform unitary operations to restore her/his target state $|\psi\rangle_{b_1b_2}$ and $|\psi\rangle_{a_1a_2}$.

Step 3. To restore Bob's state, we introduce an auxiliary particle $|0\rangle_{A_4}$. We perform a Toffoli gate with A_2, A_3 as the control qubits and $|0\rangle_{A_4}$ as the target bit. The diagram is shown in Fig.4.

3 Comparison

It's obvious that efficiency is very important factor in teleportation. In this paper, we define the efficiency η as

$$\eta = \frac{c}{p+q+t}, \tag{7}$$

where c indicates the number of the total particles which Alice and Bob want to teleport, p indicates the number of particles of the quantum channel, q indicates the number of the classical bits which are utilized, and t indicates the number of the auxiliary particles in the protocol.

In the first protocol, the efficiency is

$$\eta = \frac{4}{6+4+0} = 40\%.$$

To prove this improvement of the efficiency in this protocol, some protocols are introduced for comparison. For example, Ref. [21] is a BQT protocol of teleporting two and two qubits via six-qubit entangled state which is the same as this protocol. The efficiency of it is

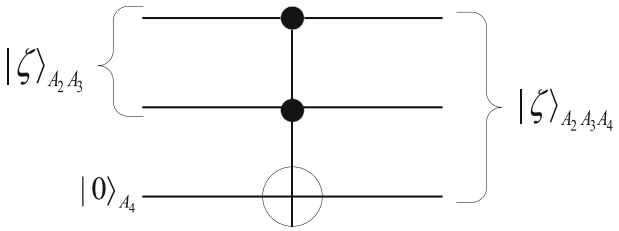

Fig. 4 The diagram of restore Bob's state by a Toffoli gate

Table 3 Comparison of different protocols with our protocols in efficiency

Protocol	Teleported state of Alice	Teleported state of Bob	Quantum channel	Efficiency (η%)
Ref. [22]	Two-qubit	Two-qubit	Eight-qubit	25
Ref. [11]	Two-qubit	Single-qubit	Five-qubit	30
Ref. [7]	Three-qubit	Two-qubit	Nine-qubit	18.52
Ref. [21]	Two-qubit	Two-qubit	Six-qubit	33.3
First of our protocol	Two-qubit	Two-qubit	Six-qubit	40
Second of our protocol	Two-qubit	Three-qubit	Six-qubit	45.5

$$\eta = \frac{4}{6+6+0} = \frac{1}{3} \approx 33.3\%.$$

From this example, we can see the efficiency of it is higher than the protocol in the same situation. More comparisons are shown in Table 3.

In the second protocol, the efficiency is

$$\eta = \frac{5}{6+4+1} = \frac{5}{11} \approx 45.5\%.$$

Ref. [7] is a BQT protocol of teleporting three and two qubits via nine-qubit cluster state, the number of particles which are transmitted is the same as this scheme. The efficiency of it is

$$\eta = \frac{5}{9+18+0} = \frac{5}{27} \approx 18.52\%.$$

We can see a lot of improvement in efficiency from the protocol of the reference to our protocol. The types of the protocols in Table 3 are all BQT.

4 Conclusion

In this paper, we propose two BQT protocols of two-two and two-three qubit, respectively. By utilizing GHZ measurement of three particles and unitary operations in our first protocol, we improve the efficiency of teleportation compared with other protocols. In the second scheme, we perform Toffoli gate on Bob's state to transform three-qubit state into two-qubit state, and perform Toffoli gate again to restore Bob's state finally. It's obviously that the efficiency of it also has improved a lot. We wish our schemes can be achieved experimentally in the future. And our protocols are all in a noiseless channel, we will consider how to achieve our schemes in noisy channels in the next step.

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